Research on adaptive null widening for plain array

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Abstract

To counter sidelobe nonstationary interference, the mismatch between training snapshots and applied data must be solved. Limited by the computational costs, it is impossible to update the weights frequently. To solve this problem, null widening methods based on covariance matrix tapering has attracted many attentions. However, existing methods do not fit the real 2-D plain array. In this paper, a null widening method for plain array based on spatial virtual interference cluster are derived. Besides, the degrees of freedom consumed by the null widening in plain array is preliminarily analyzed, and the upper bound of the degrees of freedom is obtained. The simulation results verify that the proposed method can effectively widen the adaptive null, and the derived degrees of freedom can be viewed as a guidance for the partial adaption system, which will promote its application in engineering.

Array Signal Model

A uniform plain array is assumed in the y-z plane, as shown in Fig. 1.

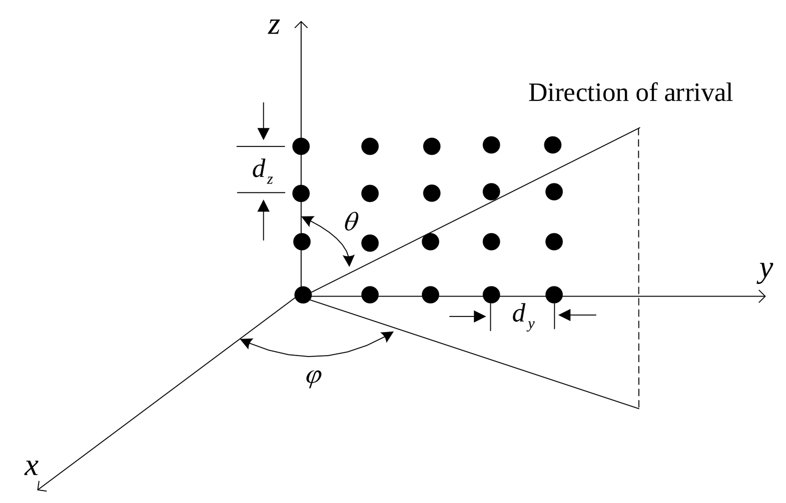

Fig.1 Diagram of plain array

For the case of multiple incoherent interference sources, the received signal $\mathbf{r}(t)$ can be further modeled as

$$\mathbf{r}(t)=\sum_{j=1}^{J}s_j(t)\cdot\mathbf{a}(\theta_j,\varphi_j)+\mathbf{n}(t)$$

where, $\mathbf{a}(\theta,\varphi)=\mathbf{a}_z(\theta,\varphi)\otimes\mathbf{a}_y(\theta,\varphi)$, $\otimes$ represents the Kronecker product. $\mathbf{a}_z(\theta,\varphi)$ and $\mathbf{a}_y(\theta,\varphi)$ are

$$\mathbf{a}_z(\theta,\varphi)=\left[1,\exp(j\frac{2\pi}{\lambda}d_z u_z),\cdots,\exp(j\frac{2\pi}{\lambda}(N-1)d_z u_z)\right]^T$$

$$\mathbf{a}_y(\theta,\varphi)=\left[1,\exp(j\frac{2\pi}{\lambda}d_y u_y),\cdots,\exp(j\frac{2\pi}{\lambda}(M-1)d_y u_y)\right]^T$$

where, $u_z=\cos\theta$, $u_y=\sin\theta\sin\varphi$.

MVDR beamformer is generally used. With K snapshots $\mathbf{r}_k(t)$, the estimated covariance matrix is

$$\hat{\mathbf{R}}=\frac{1}{K}\sum_{k=1}^{K}\mathbf{r}_k(t)\mathbf{r}_k^H(t)$$

Then, the adaptive weight can be obtained,

$$\boldsymbol{\omega}=\frac{\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta_0,\varphi_0)}{\mathbf{a}^H(\theta_0,\varphi_0)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta_0,\varphi_0)}$$

where, θ_0 and φ_0 are the elevation and azimuth angle corresponding to the direction of the beam center, respectively.

Null Widening for Plain Array

The null widening method for two-dimensional plain array is deduced from the perspective of spatial virtual interference cluster. It is assumed that there are dense incoherent interference sources in the space domain, which are distributed around the actual interference source with the same power, and the distribution range is $[-W_y/2,W_y/2]$ and $[-W_z/2,W_z/2]$, respectively. The composition of the ideal covariance matrix can be obtained,

$$\hat{\mathbf{R}}\simeq\sum_{j=1}^{J}\sigma_j^2(\mathbf{a}_{z,j}\mathbf{a}_{z,j}^H)\otimes(\mathbf{a}_{y,j}\mathbf{a}_{y,j}^H)+\sigma_n^2\mathbf{I}$$

According to Mailloux's widening analysis, it can be concluded that the widening process of the plain array can be expressed as the Hadamard product,

$$\tilde{\mathbf{R}}=\hat{\mathbf{R}}\odot\mathbf{T}$$

The widening matrix T reflects the effects of spatial virtual interference cluster in both *y*- and *z*- dimensions, and the element of $\mathbf{T}_{MN\times MN}$ in the α-th row and the β-th column can be written as

$$[\mathbf{T}]_{\alpha,\beta}=\mathrm{sinc}\left((m_z-n_z)d_zW_z/\lambda\right)\cdot\mathrm{sinc}\left((m_y-n_y)d_yW_y/\lambda\right)$$

where $m_z=\lfloor\alpha/M\rfloor+1$, $m_y=\alpha\bmod M$, $n_z=\lfloor\beta/M\rfloor+1$, $n_y=\beta\bmod M$.

With the widening covariance matrix $\tilde{\mathbf{R}}$, beamforming can effectively widen the signal received by the plain array and suppress the non-stationary interference in space.

The plain array can be modeled as the coupling of two mutually perpendicular linear arrays.

$$\tilde{\mathbf{R}}=\hat{\mathbf{R}}\odot\mathbf{T}=\hat{\mathbf{R}}\odot(\mathbf{T}_z\otimes\mathbf{T}_y)=\sum_{j=1}^{J}\sigma_j^2(\mathbf{a}_{z,j}\mathbf{a}_{z,j}^H\odot\mathbf{T}_z)\otimes(\mathbf{a}_{y,j}\mathbf{a}_{y,j}^H\odot\mathbf{T}_y)+\sigma_n^2\mathbf{I}$$

where $[\mathbf{T}_z]_{m_z,n_z}=\mathrm{sinc}\left((m_z-n_z)d_zW_z/\lambda\right)$, $[\mathbf{T}_y]_{m_y,n_y}=\mathrm{sinc}\left((m_y-n_y)d_yW_y/\lambda\right)$.

The perturbation widening of the whole covariance matrix can be equivalent to perturbation of two sub-covariance matrices respectively.

For a uniform linear array, the minimum number of degrees of freedom *D* is,

$$D\geq\left\lceil\frac{BT_\theta}{\pi}+1\right\rceil$$

where, *B* is the bandwidth, and T_θ is the time aperture.

For uniform rectangular plain array,

$$\tilde{\mathbf{R}}=(\mathbf{a}_z\mathbf{a}_z^H\odot\mathbf{T}_z)\otimes(\mathbf{a}_y\mathbf{a}_y^H\odot\mathbf{T}_y)=\tilde{\mathbf{R}}_z\otimes\tilde{\mathbf{R}}_y$$

The eigenvalue of $\tilde{\mathbf{R}}_z$ is $\lambda_{z,1}>\lambda_{z,2}>\cdots>\lambda_{z,N}$, and the eigenvalue of $\tilde{\mathbf{R}}_y$ is $\lambda_{y,1}>\lambda_{y,2}>\cdots>\lambda_{y,M}$. Then, *MN* eigenvalues of $\tilde{\mathbf{R}}$ are

$$\left[\lambda_{z,1}\quad\lambda_{z,2}\quad\cdots\quad\lambda_{z,N}\right]^T\otimes\left[\lambda_{y,1}\quad\lambda_{y,2}\quad\cdots\quad\lambda_{y,M}\right]^T$$

where, D_y and D_z are the degrees of freedom required for each dimension. It is obvious that the sum of the first D_z eigenvalues of *M* groups can guarantee the proportion of 99.99%. Therefore, for a uniform rectangular plain array, a preliminary choice of degrees of freedom can be obtained,

$$\tilde{D}\geq\min\left\{MD_z,ND_y\right\}$$

Simulation Results

The simulation parameters are given in Table I.

Table I. SIMULATION PARAMETERS

Number of array elements	20×20
System frequency	3GHz
Array distance	half of the wavelength
JNR	50dB
Direction of beamform	azimuth angle: 0° elevation angle: 90°
Direction of interference	azimuth angle: 33° elevation angle: 45°
Widening width index	0.1

Fig. 2 and 3 show the three-dimensional pattern with null widening.

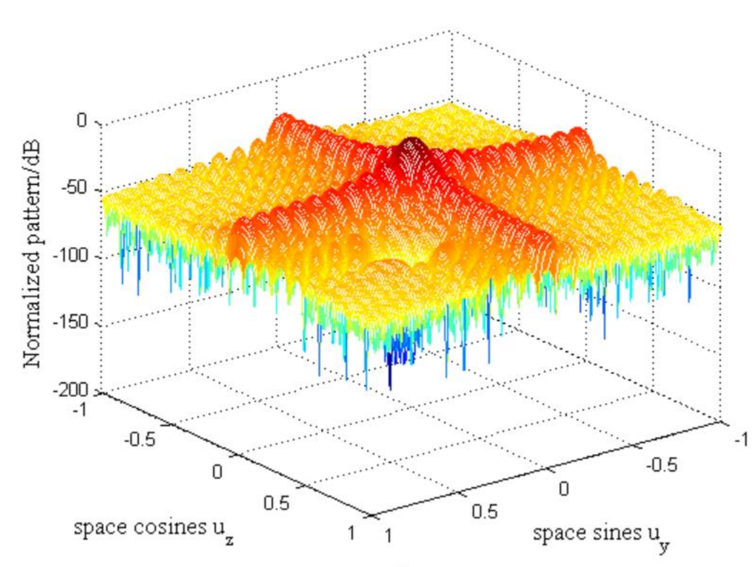

Fig.2 Widening pattern

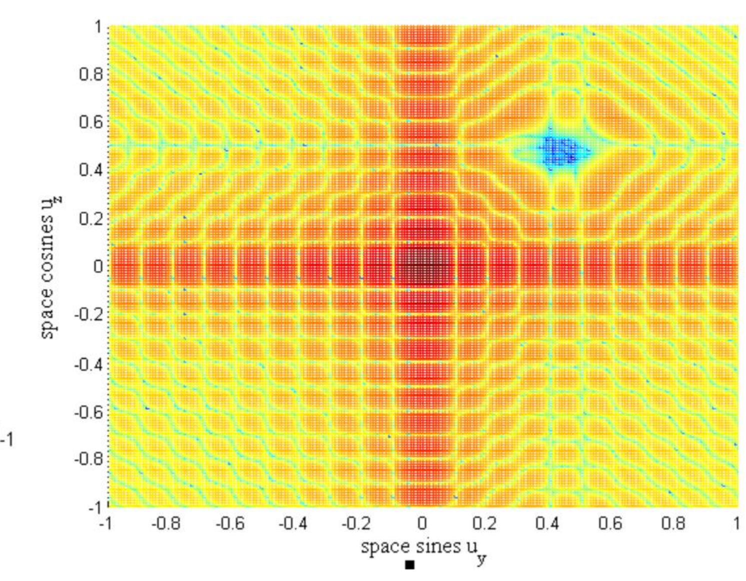

Fig.3 Widening pattern (overlook)

The eigenvalue distribution of the widening covariance matrix is presented in Fig. 4, where eigenvalues are arranged in descending order. The first 26 eigenvalues show much larger. The result calculated will be greater than this result, the validity of the freedom selection method can be guaranteed.

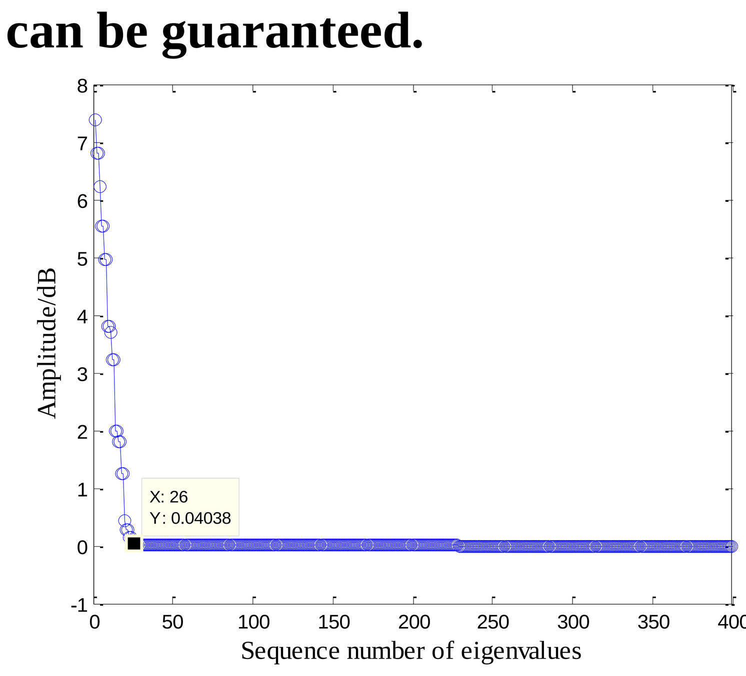

Fig.4 Distribution of the eigenvalues

Summary

The traditional null widening method is extended to the more practical uniform plain array in engineering, and a method to calculate the lower bound of the consumed degree of freedom is given. The simulation results show that the proposed method can effectively widen the null in the adaptive pattern of the plain array, providing guidance for the partial adaption system.

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