

# Super-directive Scattering of Optimized Multi-layered Dielectric Cylinder by Differential Evolution Algorithm

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## Abstract

Superdirective antenna can realize high channel capacity of multiple-input multiple-output system (MIMO). The optimization of superdirective scattering by a novel weighting optimizing goal and Dirac-Delta function model is proposed. Both main-lobe and side-lobe characteristics of the lobe are optimized. Compared with the traditional optimization method, which only takes the main-lobe characteristic into consideration, the directivity has improved by 50% (from 6.0 to 9.0). Meanwhile, the radius of the scattering object is only  $0.1\lambda_0$ , which indicates this antenna is a super-directive antenna.

## Introduction

Superdirective antenna is a kind of antenna with higher directivity compared with other antennas with similar electric dimension. The high directivity of superdirective antenna has drawn an increasing attention to its miniaturization characteristic. Unlike traditional antennae, the directivity of a superdirective antenna is not strictly related to the dimension of antenna. In fact, it is only related to the number of modes excited in the antenna and the proportion of each mode. And that implies superdirectivity is able to be realized with no regardness of antenna size. The superdirective antenna based on the multi-layer dielectric cylinder structure can achieve highly directional radiation under the condition that the physical size is much smaller than the wavelength, and it has important research value in the fields of voice communication, optical imaging and electronic bioengineering. However, the current optimization method of superdirective antenna is single and the optimization efficiency of the optimization algorithm is poor, so it is difficult to meet the requirements of future development.

This paper is expected to provide a solid theoretical basis for target detection technology and low-noise voice communication technology in complex environment, also provide a reliable research means for exploring physical landscape of sub-wavelength scale, and provide technology accumulation for potential applications such as interactive bio-electrical signal processing in the field of electronic bioengineering.

## The Scattering Properties of Dielectric Cylinder

An MLS is chosen as the excitation source. In this case,  $H_z^{MLS}$  can be written as

$$H_z^{MLS} = -I \frac{\omega \varepsilon_r r_0}{4} \begin{cases} \sum_{n=0}^{\infty} \tau_n J_n(k_0 \rho) H_n^{(2)}(k_0 \rho_s) \cos \Phi & \rho \leq \rho_s \\ \sum_{n=0}^{\infty} \tau_n J_n(k_0 \rho_s) H_n^{(2)}(k_0 \rho) \cos \Phi & \rho \geq \rho_s \end{cases} \quad (1)$$

where  $I$  is the current amplitude of the MLS source,  $\omega = 2\pi f$  and  $f$  is the frequency.  $k_i = \omega \sqrt{\varepsilon_i \mu_i}$  is the wave number for  $i$ th layer.  $|\rho - \rho_s| = \sqrt{\rho^2 + \rho_s^2 - 2\rho\rho_s \cos(\varphi - \varphi_s)}$ , here  $\tau_n$  is the Neumann Number,  $\tau_n = 1$  for  $n = 0$  and  $\tau_n = 2$  for other cases.  $J_n$  represents the  $n$ th-order of Bessel function and  $H_n^{(2)}$  is the  $n$ th-order of Hankel function of the second kind, which represents an outgoing radiation.

$\varepsilon_i$  is the relative permittivity of Layer  $i$ .  $r_i$  ( $i = 0, 1, \dots, N$ ) is the radius at the boundaries between adjacent layers. The relative permeability of each layer  $\mu_i$  is 1. Layer 0 is the free space. The location of the MLS is  $(\rho_s, \varphi_s)$  in cylindrical coordinate  $(\rho, \varphi, z)$  and the observation point is written as  $(\rho, \varphi)$ .

$$E_\rho = \frac{1}{j\omega \varepsilon} \frac{\partial H_z}{\partial \varphi}, \quad E_\varphi = -\frac{1}{j\omega \varepsilon} \frac{\partial H_z}{\partial \rho} \quad (2)$$

The boundary conditions are

$$H_z^i(\rho = r_i, \varphi) = H_z^{i+1}(\rho = r_i, \varphi) \quad (4)$$

$$E_\varphi^i(\rho = r_i, \varphi) = E_\varphi^{i+1}(\rho = r_i, \varphi), \quad i = 0, 1, \dots, N \quad (5)$$

$$M_n C_n = \Lambda_n \quad (6)$$

where

$$C_n = \begin{bmatrix} A_n^1 \\ A_n^2 \\ B_n^2 \\ \vdots \\ A_n^N \\ B_n^N \\ A_n^0 \end{bmatrix}, \quad \Lambda_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \varepsilon_0 J_n(k_0 r_N) H_n^{(2)}(k_0 \rho_s) \\ k_0 J_n'(k_0 r_N) H_n^{(2)}(k_0 \rho_s) \end{bmatrix} \quad (7)$$

The boundary conditions can be expressed as the form of matrix. The unknown coefficients  $C_n$  can be calculated once the radius, permittivities and permeabilities of each layer and the incident field are all known. Then the field of each layer can be obtained. The definition of directivity of the far field  $D(\varphi)$  can be expressed as

$$D(\varphi) = \frac{\left| \sum_{n=0}^{\infty} j^{-n} \tau_n [J_n(k_0 \rho_s) + A_n^0] \right|^2}{\sum_{n=0}^{\infty} \tau_n |J_n(k_0 \rho_s) + A_n^0|^2} \quad (8)$$

## Optimization Goal

The final goal is to maximize the directivity. The theoretical maximum value of directivity

$D_{max}(\varphi) = \sum_{n=0}^{\infty} \tau_n = 2N + 1$ . The fitness function of  $D_{max}$  is called  $f_1$ , which is written as  $\frac{\|D_{2D}(\varphi = \varphi_0) - (2N + 1)\|_2^2}{2N + 1}$ . According to the Dirac-Delta function, the distribution of side-lobe

is also fixed once the number of dielectric layer  $N$  is fixed. Due to this, the number and maximum level of side-lobes can be also taken into consideration during the optimization process. The number of side-lobes should be maximized to realize highest directivity. The maximum of side-lobe number is equal to the number of solution of  $1 + 2 \sum_{m=1}^N \cos(m\varphi) = 0$ . The solution is  $\varphi = \frac{2k\varphi}{2N + 1}$ ,  $k = 1, 2, \dots, 2N$ , so the maximum of side-lobe number is  $2N$ . The fitness function of

$N_{SL}$  is called  $f_2$ ,  $f_2 = \frac{\|N_{SL}(D_{2D}(\varphi)) - 2N\|_2^2}{2N}$ .  $N_{SL}$  is the number of side-lobes. When  $N_{SL} = 2N$ ,  $f_2 = 0$ , when  $N_{SL} = 0$ ,  $f_2 = 1$ . The maximal level of side-lobes should be also considered to realize highest directivity. The maximal level of side-lobes is written as  $M_{SL}$ . The fitness function of  $M_{SL}$  is called  $f_3$ ,  $f_3 = \frac{\|L_{SL}(D_{2D}(\varphi)) - M_{SL}\|_2^2}{M_{SL} S(L_{SL}(D_{2D}(\varphi)))}$ .  $L_{SL}$  is the level of side-lobes whose level are above  $M_{SL}$ .  $M_{SL}$  is the theoretical maximal level of side-lobes.  $S(L_{SL}(D_{2D}(\varphi)))$  is the number of lobes whose side-lobe level are above  $M_{SL}$ . When  $S(L_{SL}(D_{2D}(\varphi))) = 0$ ,  $f_3 = 0$ , when  $S(L_{SL}(D_{2D}(\varphi))) = 2N$ ,  $f_3 = 1$ .

$f_1$ ,  $f_2$  and  $f_3$  are all normalized so that their influence in fitness function are the same. If normalization is not employed, the fitness function may increase abnormally due to the unlimited value of  $f_1$ ,  $f_2$  and  $f_3$ . That leads to incorrect optimization result. The final fitness function  $f$  is written as  $f = w_1 f_1 + w_2 f_2 + w_3 f_3$ , where  $w_1$ ,  $w_2$  and  $w_3$  are the weight coefficients. Here  $w_1 = 1$ ,  $w_2 = 5$ ,  $w_3 = 1$  according to our experience. The weight coefficients helps tweak the priority of each part of the fitness function. The value of  $w_i$  ( $i = 1, 2, 3$ ) has been optimized in order to speed up the optimization process.

## Results

Compared with the traditional optimization method, which only takes the main-lobe characteristic into consideration, the directivity has improved by 50% (from 6.0 to 9.0). Meanwhile, the radius of the scattering object is only  $0.1\lambda_0$ , which indicates this antenna is a super-directive antenna.

## Conclusions

In this paper, a novel fitness function is proposed and a significant improvement of the final directivity is realized. This improvement indicates efforts on the construction of fitness function is able to increase the searching capacity of algorithms.

## References

[1] Constantine A. Balanis. *Advanced Engineering Electromagnetics, 2nd Edition*. Wiley, 2012.