

## Introduction

MIMO radar, which has multiple antennas at the receiver and transmitter, has attracted world-wide attention for the past decades. A MIMO array can be equivalent to a virtual array whose virtual array aperture is larger than the actual physical array aperture [1]-[3]. Therefore, it can increase the degrees of freedom (DOFs) and improve the angular resolution [4]-[5].

In practical applications, the design of MIMO antenna array is a key technology of the millimeter-wave MIMO radar and the configuration of the array directly affects the angular resolution. MIMO array design needs to make the virtual array aperture large and ensure the performance. A larger virtual array aperture usually requires the virtual array to be sparse so that both transmitting and receiving arrays need to be sparse [8]. For the design of sparse array, the spacing of array elements is commonly larger than half wavelength, which may result in a bad radiation pattern. Therefore, it is necessary to optimize the array design. The optimization of the antenna positions is a highly non-linear problem which was solved by employing genetic algorithms (GAs) in recent years [6]- [7]. The GA is a kind of parallel, efficient, and global search optimization method, and plenty of researches focused on the design of sparse array based on the GA. But few of them noticed the practical problems of radar system design, such as the antenna size and the feeder length.

Hence, in this paper, a design method of MIMO array based on GA is proposed by fully considering the practical problems of the radar system design. In this method, we restrict the maximum array aperture according to the size of the given PCB board, constrain the minimum array spacing according to the actual size of the antenna, and limit the range of the array element positions according to the chip position. The array configuration optimized by the proposed method has low sidelobe pattern and realize the resolution requirements. Therefore, by considering the above actual, the optimized MIMO array can be effectively applied to the actual MIMO radar system.

## Design Method

### A. MIMO antenna array and virtual array

MIMO radar transmits orthogonal signals from each element at the transmitter, receives the target echo signal simultaneously at the receivers, and then separates the transmitted signals through matched filters [9]. If the radar system transmits  $M$  orthogonal signals through  $M$  transmitting antennas and receives echo signals through  $N$  receiving antennas, then  $M$  matched filters are required to separate these signals. In this way, it can be equivalent as  $MN$  transceiver channels. Assuming that transmitting and receiving antennas are all distributed at the integral multiple of  $\lambda/2$ , the position set of transmitting and receiving elements are, respectively, formulated as follows

$$Tx = [x_{t1}, x_{t2}, x_{t3}, \dots, x_{t(M-1)}, x_{tM}], \quad x_{t1} = 0$$

$$Rx = [x_{r1}, x_{r2}, x_{r3}, \dots, x_{r(N-1)}, x_{rN}], \quad x_{r1} = 0$$

where  $x_{t2} < x_{t3} < \dots < x_{tM}$ ,  $x_{r2} < x_{r3} < \dots < x_{rN}$  and they are all integers in the unit of  $\lambda/2$ . According to the technical theory of the MIMO radar, the position set of the equivalent array is

$$TRx = \{x_{ti} + x_{rj}\}; i=1, \dots, M, j=1, \dots, N$$

The position difference of each element in the equivalent virtual array is given as follows

$$Zx = \{(x_{tm1} + x_{rn1}) - (x_{tm2} + x_{rn2})\}$$

where  $m1, m2 \in [1, M]$ ,  $n1, n2 \in [1, N]$ .

### B. Genetic algorithms

GA simulates the process of biological evolution and is a parallel, efficient, and global search optimization method. It is suitable for parallel processing, especially for solving complicated and nonlinear problems. GA generally starts from an initial population and evaluates each individual according to the fitness function. Then through the individual selection, information crossover, and variation, a new generation of population is produced. The population evolves from generation to generation until it reaches a predetermined value or genetic algebra.

### C. MIMO antenna array design

By using the GA, the MIMO array with  $M$  transmitting antennas and  $N$  receiving antennas is designed as follows:

Step1: Determine the array aperture according to the resolution requirements of the MIMO radar and PCB board size, the aperture of transmitting array and receiving array is

$$L_t = (L_M - 1) \times \lambda/2, \quad L_r = (L_N - 1) \times \lambda/2,$$

where  $L_M$  and  $L_N$  are the number of full elements of transmitting array and receiving array, respectively.

Step2: Determine the minimum array spacing  $d_c$  according to the physical size of antenna elements in practical application.

Step3: According to the number of chips  $K$  and the aperture of the antenna array, constrain the distribution range of each antenna element. For example, both the transmitter and receiver apertures are divided into  $K$  sub-intervals, and the position of each element corresponding to each chip is constrained within a specific interval. Each interval can be expressed as

$$[0, L_{sub1}], [L_{sub1} + 1, 2L_{sub1}], \dots, [(K-1)L_{sub1} + 1, L_M]$$

$$[0, L_{sub2}], [L_{sub2} + 1, 2L_{sub2}], \dots, [(K-1)L_{sub2} + 1, L_N]$$

where  $L_{sub1} = \lceil (L_M - 1) / K \rceil$ ,  $L_{sub2} = \lceil (L_N - 1) / K \rceil$ , and  $\lceil x \rceil$  represents an upper round integer of  $x$ .

Step4: The GA is applied to optimize the MIMO array design. According to the parameters and constraints obtained in step 1), the population is generated and each individual represents an array arrangement. The  $g$ -th populations of the transmitting array and the receiving array are denoted as  $F_{t,g}, F_{r,g}$ . They are the vectors with the first and last elements being 1, expressed as follows

$$F_{t,g} = (1, \underbrace{0, 1, \dots, 0}_{\text{the number is } L_M - 2, \text{ the number of '1' is } (M-2)}, 1)$$

$$F_{r,g} = (1, \underbrace{1, 0, \dots, 0}_{\text{the number is } L_N - 2, \text{ the number of '1' is } (N-2)}, 1)$$

Herein, the application process of GA is as follows

a. Determine the population size  $N_p$ , number of generation  $G$ , crossover rate  $P_c$ , mutation rate  $P_m$ . First, the initial population is generated as follows

$$\begin{cases} A_i = randi([2, L_M - 1], M - 2, 1) \\ B_i = randi([2, L_N - 1], N - 2, 1) \\ F_{ti,0} = zeros(L_M, 1); \quad F_{ri,0} = zeros(L_N, 1) \\ F_{ti,0}(1) = F_{ti,0}(L_M) = 1; \quad F_{ri,0}(1) = F_{ri,0}(L_N) = 1 \\ F_{ti,0}(A_i(j)) = 1, \quad F_{ri,0}(B_i(j)) = 1 \end{cases}$$

The  $zeros(m, n)$  means to generate an  $m$ -by- $n$  matrix of zeros, the  $randi([M_1, M_2], m, n)$  means to generate an  $m$ -by- $n$  array containing integer drawn from the discrete uniform distribution on the interval  $[M_1, M_2]$ , and  $i = 1, 2, \dots, N_p$ , which means the  $i$ -th individual.

b. Select the dominant individuals to be retained in the next generation. We use roulette selection method to determine the possibility of offspring retention by the proportion of each individual fitness. The fitness of the  $i$ -th individual is  $fit_i$ , then its probability of being selected is

$$p_i = fit_i / \sum_{i=1}^{N_p} fit_i, \quad i = 1, 2, \dots, N_p$$

The individual fitness function is the maximum sidelobe level, expressed as follows

$$fit_i = MSL_{L(i,g)} = \max_{\theta \in S} \left\{ \frac{F(\theta)_{(i,g)}}{\max(F(\theta)_{(i,g)})} \right\}$$

where the  $F(\theta)_{(i,g)}$  is the radiation pattern function of the  $i$ -th individual in the  $g$ -th generation, and  $S$  is the sidelobe interval.

c. Pair the  $F_{t(2k-1),g}$  with  $F_{t2k,g}$  and  $F_{r(2k-1),g}$  with  $F_{r2k,g}$ , and swap some of their genes for each pair with the crossover rate  $P_c$  and the  $k = 1, 2, \dots, N_p/2$ . For each pair of crossover individuals, some locations are randomly selected as crossover points and then a random number on the interval  $[0,1]$  is generated to compare with the crossover rate to determine whether to crossover. And new individuals are formed after the exchange of genes.

d. Mutate some genes with a probability of mutation rate  $P_m$ . For the  $i$ -th individual in the  $g$ -th generation, when  $n_1 = 2 - L_M - 1$ , the random number  $r_1$  between 0 and 1 is generated. Similarly, when  $n_2 = 2 - L_N - 1$ , the random number  $r_2$  between 0 and 1 is generated. If  $r_1 < P_m$ , the  $F_{ti,g}(n_1)$  mutate. Also, if  $r_2 < P_m$ , the  $F_{ri,g}(n_2)$  mutate.

e. Guarantee each individual in the new population meet the constraints in steps 2) and 3). If not, the individual will be re-mutated until it can meet these constraints.

f. Calculate the fitness of all individuals in the new generation, and the individual with the largest fitness is selected as the best and be reserved to the next generation.

g. Determine whether the termination condition of the GA is met. If so, the optimal individual will be output. If not, the next generation will continue to be generated by going back to step b.

## Experimental Results

A MIMO array with 9 transmitting antennas and 12 receiving antennas would be designed and the angular resolution is set as  $0.55^\circ$ . Calculating the required array aperture of the virtual array and then determine the maximum aperture of the transmitting array and receiving array that meet the requirements. Herein, we set  $L_M = 105$ , and  $L_N = 104$ . Assuming that the antenna element size is about  $L = 4 \times \lambda/2$ , then the minimum array element spacing  $d_c$  is 4.

Set the population size is  $N_p = 50$ , the number of generations  $G = 1000$ , crossover rate  $P_c = 0.8$ , and mutation rate  $P_m = 0.05$ . The fitness evolution curve is plotted in Fig. 1, and the maximum fitness is about 15.76 dB. The design results of the MIMO array are shown in Fig. 2, where the position sets of transmitting and receiving array elements are, respectively,  $Tx = [0; 13; 30; 36; 43; 55; 78; 90; 104]$  and  $Rx = [0; 4; 15; 24; 36; 44; 48; 53; 75; 81; 88; 103]$ . Fig. 3 illustrates the position distribution diagram of the equivalent virtual array elements, the maximum virtual array aperture is  $L_v = 207 \times \lambda/2$ , which meets the requirement of the array design. Fig. 4 shows the distribution of position difference of virtual array elements, the maximum continuous value of array element position difference is 181, which is really good for a sparse array. Next, we simulate the beampattern of the equivalent virtual array with the steering angle being  $0^\circ$ , as shown in Fig. 5. As can be seen, the mainlobe is quite narrow and its angular resolution is able to realize the requirement. In addition, the sidelobe is below -15.76 dB. The low sidelobe is able to guarantee a good target detection performance.

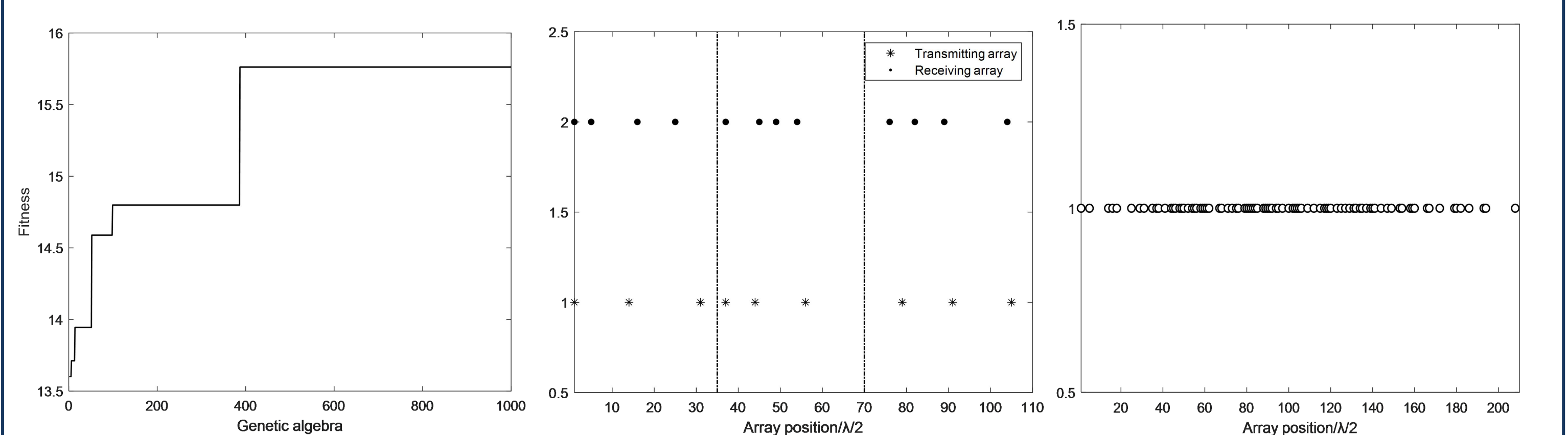


Fig. 1. Fitness evolution curve. Fig. 2. Position distribution of array elements. Fig. 3. Equivalent virtual array elements.

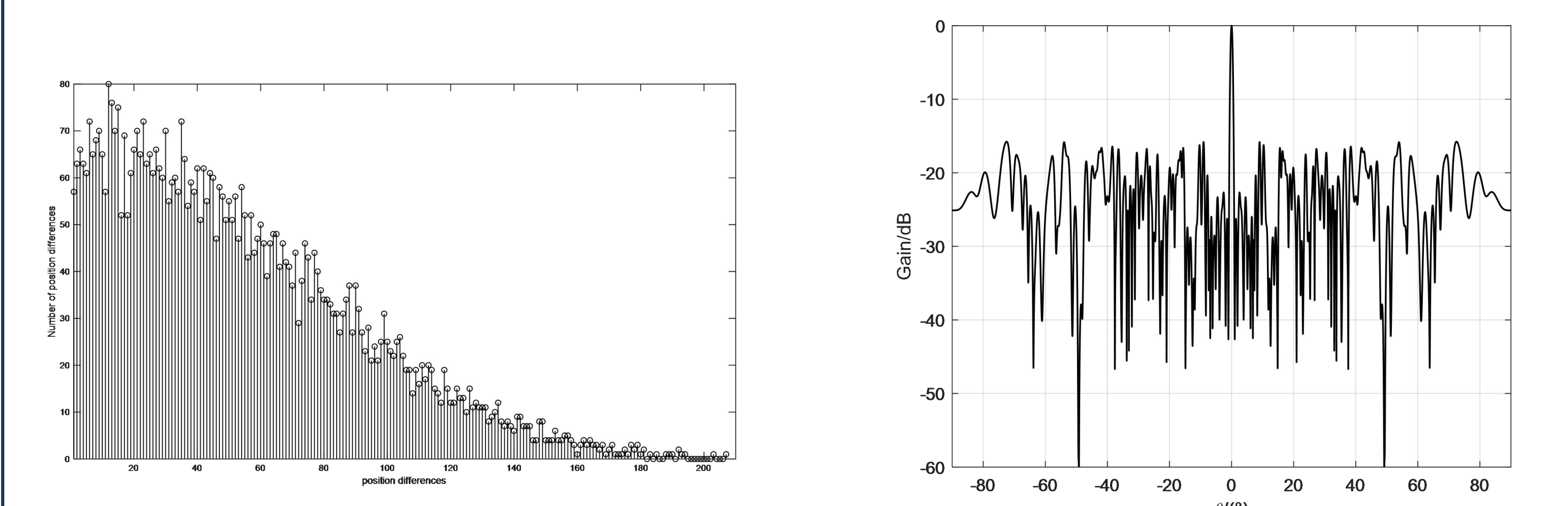


Fig. 4. Position difference distribution of virtual array elements. Fig. 5. Radiation pattern of the equivalent virtual array.